

## Assessment of the cognitive visualization process used in learning geometry<sup>1</sup>

## Évaluation du processus cognitif de visualisation utilisé dans l'apprentissage de la géométrie

## Avaliação do processo cognitivo de visualização utilizado na aprendizagem da geometria

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*Assessment of cognitive processes involved in the learning of geometry is a real challenge to understand difficulties learners faced. This article investigates the assessment of cognitive processes through two studies on the spatial visualization ability, a first in plane geometry and a second in 3D geometry. The first study uses a paper-and-pencil test used to assess the ability to visualize plane figures at the end of primary education. The second employs a test that uses virtual material, as an alternative to paper-and-pencil test, to assess 3D spatial visualization with primary and secondary learners. After a description of the two assessment tools, the aim of this article is to take a critical look at the evaluation of cognitive processes by presenting the limits relating to these tools and by identifying the alternatives offered notably with the emergence of new technologies.*

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Mots clés : évaluation informatisée, évaluation papier-crayon, géométrie, processus cognitifs, visualisation

*L'évaluation de processus cognitifs impliqués dans l'apprentissage de la géométrie constitue un réel défi pour comprendre de nombreuses difficultés d'apprentissage dans ce domaine. Cet article investigate l'évaluation de processus cognitifs au travers de deux études portant sur l'habileté de visualisation spatiale. Centrée sur la géométrie plane, la première étude utilise une épreuve de type papier-crayon, réalisée auprès d'élèves en fin d'enseignement primaire, pour évaluer les capacités de visualisation des figures. Centrée sur la géométrie tridimensionnelle, la seconde étude implique une épreuve évaluant la visualisation spatiale réalisée avec du matériel virtuel comme alternative aux épreuves papier-crayon, auprès d'élèves du primaire et du secondaire inférieur. En analysant notamment certaines productions d'élèves, l'objectif de cet article est également de poser un regard critique sur l'évaluation de processus cognitifs en présentant les limites relatives aux évaluations présentées et en identifiant des alternatives offertes par l'émergence des nouvelles technologies, entre autres.*

PALAVRAS-CHAVES: avaliação informatizada, avaliação papel e lápis, geometria, habilidades espaciais, processo cognitivo, visualização

*A avaliação dos processos cognitivos envolvidos na aprendizagem da geometria representa um grande desafio para compreender as diversas dificuldades de aprendizagem neste domínio. Este artigo investiga a avaliação de processos cognitivos por meio de dois estudos focados nas habilidades de visualização espacial. Centrada na geometria plana, o primeiro estudo utiliza uma prova do tipo papel e lápis, aplicada a alunos do final do ensino básico, para avaliar as suas habilidades de visualização de figuras. Focando na geometria tridimensional, o segundo estudo envolve uma prova de avaliação da visualização espacial realizada com material virtual como alternativa às provas tradicionais de papel e lápis, aplicada a alunos do ensino básico e do ensino secundário. Ao analisar, especialmente, algumas produções dos alunos, o objetivo deste artigo é também adotar uma visão crítica sobre a avaliação dos processos cognitivos, apresentando as limitações das avaliações descritas e identificando alternativas oferecidas pelo surgimento das novas tecnologias, entre outras.*

## Introduction

The development of geometric competences involves many complex cognitive processes, beginning with spatial abilities (Baldy et al., 2005; Clements & Sarama, 2007; Kaur et al., 2018; Soury-Lavergne & Maschietto, 2015). Assessing these processes is a crucial pedagogical challenge that helps teachers understand and remedy learning difficulties.

In this article, we explore two studies to investigate the assessment of cognitive processes relating to spatial visualization ability in geometry learning: The first in plane geometry (2D), and the second in three-dimensional geometry (3D). Spatial ability is defined as the process by which a person mentally represents non-verbal spatial information, analyses the relationships between objects in a configuration, and performs mental operations on these objects (e.g., Eme, 2003; Loranger et al., 2000; Marchand, 2006). As this ability is key to understanding geometry (Gal & Linchevski, 2010), gaps in its acquisition may cause learning difficulties in this domain (Fabiyl, 2017).

This article explores the assessment of visualization cognitive processes by presenting the two studies and examining the assessment tools used in them, discussing the limitations of paper-and-pencil assessments and identifying the possibilities offered by emerging technologies. The first study investigates the assessment of the visualization process of plane figures: the way figures are viewed. Duval (2005) identifies two modes of figure visualization: iconic visualization and non-iconic visualization. Even though it corresponds to the intuitive way of looking at figures, the iconic visualization mode seems to explain impasses for entering into a geometric approach, which is essential, particularly during secondary education. It prevents pupils from perceiving figures as assemblages of elements with geometric properties. However, secondary pupils are expected to be able to position themselves in the non-iconic visualization mode, despite it being counter intuitive. Therefore, it appears important to be able to use assessment to determine whether pupils are capable of demonstrating

non-iconic visualization or whether they are still in iconic mode. This is necessary for both understanding certain difficulties encountered and proposing activities that teach pupils to move from iconic visualization to non-iconic visualization, as envisaged by several authors (e.g., Duroisin et al., 2020; Duval & Godin, 2005; Keskessa et al., 2007; Perrin-Glorian et al., 2013). The first study uses a paper-and-pencil test in an experimental context (Beuset & Duroisin, 2021) to identify the visualization mode of pupils at the end of primary education. We describe the limitations met when using this test, particularly difficulties in understanding the reasoning used for the final production. We then present the possibilities offered by digital assessments.

Moving away from Duval's conceptual framework, which organizes visualization according to two distinct modes, the second study focuses on the acquisition of the spatial visualization ability of 3D objects, which can be summarized as the capacity to imagine and mentally transform visually perceived 3D objects (Barisnikov & Pizzo, 2013). As this ability appears necessary for learning 3D geometry (Kaur et al., 2018), assessing it can help identify certain pupil profiles, understand difficulties encountered in learning, and propose adapted interventions (Vander Heyden et al., 2016). The field of cognitive psychology proposes many paper-and-pencil tests, including the Spatial Reasoning Instrument (Ramful et al., 2017). Nevertheless, several authors, including Hawes et al. (2015) or Rahe and Quaiser-Pohl (2022), have identified limitations in this type of test and envisage alternative solutions by asking respondents to demonstrate spatial visualization on physical 3D material rather than plane representations. The second study describes an experiment using an alternative with virtual representations of solids for assessing the ability.

In this article, we present the two studies by describing the theory behind them and the challenges associated with assessing the cognitive process. We follow with a presentation of the methodology implemented and a description of the assessment material used during the experiment. We then examine the implementation of the assessment, drawing on pupils' productions and scientific literature. Finally, we propose a conclusion that summarizes both studies with a more critical analysis of the assessment of cognitive processes in geometry and emerging and contemporary issues.

***The First Study: Assessment of the Way Figures Are Viewed in 2D Geometry***

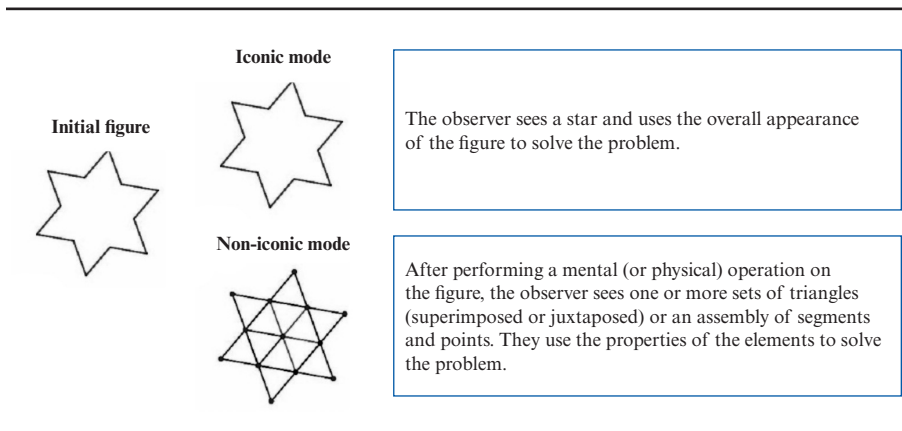
This first study uses a paper-and-pencil assessment tool to determine the visualization mode, as referred to in Duval’s theory (2005), of pupils in the final cycle of primary education. The participatory research (Beauset & Duroisin, 2021, 2023) with almost experimental design (N = 104) was conducted in French-speaking Belgium. The tool was used pre- and post-experiment to validate a pedagogical device used to develop visualization amongst pupils at the end of primary education.

***Figure Visualization: Two Opposing Ways of Seeing Figures***

According to Duval (2005), the term “visualization” evokes the way figures are viewed, upon which pupils then base their actions and reasoning. Duval (2005) identifies two opposing modes of the cognitive function of visualization: iconic and non-iconic.

In the iconic mode, people rely on perceptual observations and may also take measurements to see and identify figures. The overall appearance of the figure is then definitional (Mithalal, 2011), as the person identifies the figure and reasons to use the resemblance of the shape with a typical shape from their repertoire. As Figure 1 illustrates, in the case of iconic visualization, the observer associates the proposed drawing with a star and uses reasons by drawing on the overall shape of the star.

Figure 1  
*Iconic and non-iconic visualization*



The non-iconic mode uses mental or physical operations on the figure such as adding tracings to reorganize the figure and identify its geometric properties to enable reasoning. For Figure 1, the observer is then capable of moving beyond the overall appearance of the star by visualizing the shape as an assemblage of triangles and reasoning to use the properties of this assemblage.

Duval's theory (2005) can be linked to the modes of analysis described by Davydov (1990, cited by Venenciano et al., 2021) whose work exceeds the scope of the specific framework of visualization of geometric figures. The iconic mode is likened to so-called empirical analysis: The pupil reasons using empirical generalizations based on aspects detectable on objects, especially those directly observed through perception. In contrast, non-iconic visualization is more like theoretical analysis: The pupil relies on theoretical properties to apply theoretical generalizations to understand the internal structure of concepts. They are capable of transcending the field of perception and reaching idealization. When learning, students require support to develop their capacity to analyze problems theoretically rather than empirically, replacing empirical with theoretical generalizations.

In everyday life, people are naturally in the iconic visualization mode. They observe shapes and drawings that they attempt, at first glance, to associate with objects from their repertoire. This mode appears sufficient for geometry learning objectives in primary education, but not for secondary education. Indeed, it leads to impasses for the continued learning and entering a geometric approach (Duval, 2005). One impasse is caused by how visual elements can be deceptive, which undermines reasoning as it is based on incorrect information. Moreover, pupils in the iconic visualization mode focus on the general shape of the geometric object and its contours. They cannot exploit all the figure's properties, particularly those not associated with the contours, such as diagonal properties. It is also difficult for them to consider the elements that constitute the shape, for example, the segments forming the sides of the shape, isolated from the general shape (Mithalal, 2011). They are also unable to visually transform and reorganize the figure without distorting the shape.

Despite being counter-intuitive, the transition to non-iconic mode is therefore expected for secondary education. This mode is indispensable for understanding the formulations of essential geometric properties, particularly incidence properties, which become more meaningful (Bulf & Mathé, 2018; Duval, 2005; Duval & Godin, 2005; Perrin-Glorian &

Godin, 2018). It also facilitates the development of geometric competences such as demonstration (Barrier et al., 2014; Duval, 2005; Perrin-Glorian, 2012). Thus, Godin and Perrin-Glorian (2008) emphasize the importance of “education in changing one’s way of looking” (p. 15). Nevertheless, Beuset and Duroisin (2021) note that need for support to change perspective can vary, particularly given different paces of transition and a strong persistence of iconic visualization mode amongst certain pupils.

Given these observations, assessing a pupil’s visualization mode appears to be important for understanding certain difficulties they encounter and for providing adapted support in the transition towards their geometric view of figures.

### *Description of the Test Assessing Visualization Mode*









In order to determine pupils’ visualization mode at the end of primary education, a paper-and-pencil assessment was designed (Beuset & Duroisin, 2021). This test, planned to take no longer than 50 minutes, features eight varied figure restoration problems, drawn from or inspired by external certification tests organized in French-speaking Belgium or from previous research on the subject (e.g., Keskessa et al., 2007). In this type of problem, illustrated in Figure 2, pupils are given a model figure and are asked to reconstruct the figure starting from a prompt, a partial drawing of the figure. The eight restoration problems vary according to the type of geometric object to be restored (configuration of lines, simple figure, complex figure, figure including a circle, etc.), the type of approach necessary for solving them (exploitation of alignment properties only, use of measurement transfer, etc.) and the type of restoration (identical or with enlargement). To solve the exercises, pupils have the usual construction material: compass, set square, and ruler. They are also asked to leave traces of their constructions. Annex 1 presents the entire test.

The choice of figure restoration problems is justified by the link between development of non-iconic visualization and figure restoration activities, highlighted by many research studies (Duval & Godin, 2005; Mangiante-Orsola & Perrin-Glorian, 2014; Mathé, 2008). Pupils can use different methods to solve these problems which reveals which visualization mode they use. For example, pupils can use more perceptual observations or take measurements of the model figure, which suggests the iconic mode, or use properties observed on the initial model figure to reconstruct from the prompt, which suggests the non-iconic mode.

It is therefore the analysis of pupils' final productions, and more particularly the analysis of the pupil's tracings on the model and prompt when carrying out the restoration, which helps to determine their visualization mode. More concretely, a list of seven criteria serves to analyze pupils' productions:








- 1) The pupil has drawn on the model.
- 2) The pupil shows some useful reorganizing tracings (i.e., which reveal properties interesting for carrying out the construction) on the model.
- 3) The pupil shows some useful reorganizing tracings on the prompt.
- 4) The pupil shows all useful reorganizing tracings on the model.
- 5) The pupil shows all useful reorganizing tracings on the prompt.
- 6) The constructed figure is partially correct (in other words, it corresponds at least in part to the model figure).
- 7) The constructed figure is totally correct (in other words, it corresponds totally to the model figure).

Figure 2  
*Examples of Problems Proposed in the Test*

<p style="text-align: center;"><b>Example 1:</b></p> <p><b>REPRODUCE</b> drawing 1 exactly, using drawing 2 as a starting point. Use the instruments of your choice. Your lines must be precise. Leave the traces of your construction.</p> <div style="text-align: center;"> <p>1. </p> <p>↓</p> <p>2. </p> </div>	<p style="text-align: center;"><b>Example 2:</b></p> <p><b>REPRODUCE</b> drawing 1 exactly, using drawing 2 as a starting point. Use the instruments of your choice. Your lines must be precise. Leave the traces of your construction.</p> <div style="text-align: center;"> <p>1. </p> <p>↓</p> <p>2. </p> </div>
<p style="text-align: center;"><b>Example 3:</b></p> <p><b>REPRODUCE</b> drawing 1 exactly, using drawing 2 as a starting point. Use the instruments of your choice. Your lines must be precise. Leave the traces of your construction.</p> <div style="text-align: center;"> <p>1. </p> <p>↓</p> <p>2. </p> </div>	<p style="text-align: center;"><b>Example 4:</b></p> <p><b>REPRODUCE</b> drawing 1 exactly, using drawing 2 as a starting point. Use the instruments of your choice. Your lines must be precise. Leave the traces of your construction.</p> <div style="text-align: center;"> <p>1. </p> <p>↓</p> <p>2. </p> </div>

For each exercise, one point is awarded per criterion present in the production. Thus, each exercise is awarded a score out of 7, allowing a total score out of 56 for the entire test. A high total score relates to solutions respecting the different criteria and therefore seems to determine that the pupil is in the non-iconic visualization mode. For a concrete illustration, Figure 3 proposes the precise description of one of the test problems, namely Example 3 (Figure 2), and shows examples of pupils' productions and their interpretation. We can therefore observe if the production contains reorganizing tracings of the figure on the model or on the prompt, showing that the pupil identified and exploited the relationships and properties of the figure to solve the problem, which indicates the non-iconic mode.

Figure 3  
*Explanation of Example 3: Pupils' Productions and Interpretations*

	<p>In order to reconstruct the figure identically, it is necessary to determine the position of the center of the circle, which can be done by adding lines such as the diagonals and medians of the square to exploit the properties of these new lines (for example, the center is the point of intersection of the medians and diagonals, but is also their midpoint).</p>	
<p><b>Production example 1</b></p> <p>1. </p> <p>↓</p> <p>2. </p> <p><b>Criteria validated:</b> 1-2-3-4-5-6-7</p> <p><b>Score:</b> 7/7</p> <p>The pupil used drawings to highlight the properties of the center of the circle (located at the intersection of the square's diagonals) on the model and then reused them on the sketch to restore the figure. He successfully restored it identically, indicating he is most likely in the <b>non-iconic mode</b>.</p>	<p><b>Production example 2</b></p> <p>1. </p> <p>↓</p> <p>2. </p> <p><b>Criteria validated:</b> 3-5-6-7</p> <p><b>Score:</b> 4/7</p> <p>The pupil constructed the <b>center</b> of the circle using its properties (location at the intersection of the square's diagonals), as shown by the lines he drew on the draft paper. However, he did not draw any lines on the model to restore the figure but probably identified this property on the model without drawing it. He successfully restored the figure identically, indicating he is most likely in the <b>non-iconic mode</b>.</p>	<p><b>Production example 3</b></p> <p>1. </p> <p>↓</p> <p>2. </p> <p><b>Criteria validated:</b> 6</p> <p><b>Score:</b> 1/7</p> <p>The pupil placed the center of the circle randomly and approximately. He therefore did not use the properties of the circle, indicating he is most likely in the <b>iconic mode</b>.</p>

### ***Discussion Relating to the Assessment: Strengths, Limitations and Alternative Assessments***

The results obtained in the assessment are consistent with scientific literature, as they show that pupils at the end of primary education are *a priori* in the iconic mode and that, if offered appropriate activities, at least partial change of their visualization mode is possible (Beauset & Duroisin, 2021, 2024a).



This test was initially used with pupils in the final year of primary education (N = 104). However, with no prerequisites of geometric knowledge, as referred to by Marchand (2006), as opposed to spatial knowledge which is knowledge relating to ideal objects referring to theorized or axiomatized school content to create a coherent system. For example, in the exercises, the pupil must reproduce a drawing without having been told the names of the geometric figures. They must therefore explore the spatial properties of the drawing such as alignment properties but must not use properties from their geometric knowledge of figures. The test could therefore be used with younger classes. Moreover, since figure restoration exercises are complex problems, this test could be made more complex for use with pupils at the beginning of secondary education, and even later. In addition, this test, whether by its duration (50 minutes), its medium (paper-and-pencil test), or the method of analyzing results (verification of seven criteria on the pupil's production), seems to be adapted to the school context and could therefore be used by teachers. This requires teachers to be trained on the theme, or at least made aware of it, so they understand the utility of the assessment and can interpret the results. However, Bulf and Mathé (2018) noted a difficulty for primary teachers to understand the challenges and purposes of geometry and the cognitive functioning of pupils in the domain.

Despite these advantages, the test is limited for assessing the cognitive process of visualizing plane figures due to difficulties interpreting certain pupils' productions. Indeed, certain productions, particularly when the restoration is successful but no tracing is shown, do not indicate which visualization mode the pupil used (see Figure 4). Some pupils may have reorganized tracings by placing the ruler on the drawing but without having drawn on the paper. Their score will indicate iconic mode, yet they used non-iconic visualization. As a result, in certain cases, the scores underestimate the level of acquisition of non-iconic visualization. This limitation shows that relying solely on pupils' final productions is insufficient for

studying the complex cognitive processes involved in solving geometry problems and seems inherent to paper-and-pencil tests. This means working with pupils' final productions only, which do not always provide information on how they reasoned and used the instruments. Conversely, in some cases, pupils leave tracings on the model or prompt they did not use in their final reasoning, which influences the calculated score positively.

Note, however, that exercises involving restoration with enlargement or reduction of the size of the figure (such as Example 4 in Figure 2) are particularly interesting for determining visualization mode. In such exercises, there is less ambiguity in the interpretation of errors, making it easier to identify the approach used. For example, it is easier to identify productions where a pupil has used measurement transfer (iconic mode), when the final production, or part of it, is exactly the same size as the initial figure, therefore not respecting the enlargement.

Figure 4  
*Example of a Pupil's Production That Is Difficult to Interpret*

<p>1. </p> <p style="text-align: center;">↓</p> <p>2. </p>	<p><b>Criteria validated:</b> 6-7</p> <p><b>Score:</b> 2/7</p> <p>The pupil managed to produce an identical reproduction. However, due to the absence of lines, it is not possible to tell whether the center was positioned using properties, by taking measurements, or at random. It is difficult to determine the mode of visualization, even though the score is in favor of the iconic mode.</p>
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The scores out of 7 allow overall determination of pupils' visualization mode, but pupils with the same score are at an equal level, regardless of the validated criteria. Thus, a pupil having obtained a score of 3 out of 7 by validating criteria 1, 2, and 4 is considered to be at the same level as a pupil that validated three other criteria. To counter this limitation, analysis by criteria, rather than by score, would identify which criteria are most often validated by pupils and provide information about the difficulties they encounter.

Another limitation of the test is due to the presence of other cognitive processes likely to influence performance on the proposed plane figure visualization task. For example, executive functions can intervene in figure restoration tasks (Beauset et al., 2025). Amongst them, working memory, which temporarily processes and stores information for a few seconds

(Baddeley, 2000), is necessary for retaining the different properties observed on the model before using them in reconstruction. This is also the case with the executive function of inhibitory control, which is the supervision process blocking interference and information irrelevant to the task (Bialystok & Craik, 2022). Finally, the executive function of cognitive flexibility, defined as the capacity to move from one task to another or to adapt to a new situation (Miyake & Friedman, 2012), intervenes during the displacement of attention between the different elements constituting the geometric figure (Khatin-Zadeh, 2022). In this case, cognitive flexibility intervenes when observing the different properties that compose the complex figure to be reproduced, and during the displacement of attention between the model figure and the prompt figure when identifying the missing parts to be reproduced.

Apart from executive functions, figure restoration also requires visuo-constructive abilities, designated by Grossi and Trojan (2004, cited by Arents et al., 2012) as the capacities to correctly arrange elements amongst themselves to produce a drawing or construction. Thus, motor capacities engaged in the use of instruments also affect pupils' performances, which can be problematic when assessing certain profiles, such as pupils with dyspraxic disorders.

Other elements can also affect pupils' performances on the test. For example, a didactic contract which implies understanding of the task conditions (Duroisin et al., 2020), leading certain pupils to understand that drawing on the model is not allowed. Such a contract necessarily affects pupil's results as drawing on the model is an assessment criterion.

Some of the above limitations could be overcome by asking the pupil to describe the method used, in writing for example. Nevertheless, this requires important metacognitive and language competences for the pupil that may affect their result negatively. It also requires more substantial analytical work for the teacher. A second option, probably more adapted to the research context than to school, is filming the pupils' actions as they solve the paper-and-pencil test to capture the processes used.

A third option could be digitizing the test in an environment with virtual measurement and tracing instruments, such as the *Géo tracé* toolkit or *Instrumentpoche*, which would provide teachers with more traces of the construction. Once again, analyzing the pupils' recorded actions, as well as their final production, would help to identify the steps they used in their construction. Digital adaptation requires pupils' master the

software but seems less demanding than the motor abilities required for using geometric instruments. This type of software would also be a solution for teaching geometry to pupils with motor disabilities (Sagot, 2005) or dyspraxic disorders (Petitfour, 2015). Software replaces the fine motor skills necessary for manipulating instruments required for paper-and-pencil tests with keyboard and/or mouse manipulations, making actions faster and easier. The resulting production is also tidier than paper-and-pencil tests, which facilitates interpretation. Note, however, as Petitfour (2015) emphasizes, trial-and-error techniques in the paper-and-pencil and the digital environments are not perfectly comparable which can impact the solution strategies used, and therefore the visualization mode. For example, in the digital environment, by simply sliding the virtual ruler, it is possible to perfectly transfer an orientation of a segment to draw a segment parallel to a given segment. This type of gesture is useful and easy to implement to perfectly succeed a restoration where the prompt has the same orientation as the model, resulting in a valid production. In the paper-and-pencil environment, on the other hand, the same gesture of transferring an orientation of a line by simply sliding the ruler, often used by pupils (Beauset & Duroisin, 2021), is incorrect use of the instrument and probably results in an imperfect final production. Furthermore, this type of gesture is more likely to be attributed to the iconic visualization mode as the pupil attempts to approximately transfer the orientation of a segment without relying on the figure's properties before its reproduction (e.g., alignment properties). Hence, unless the orientation of the prompt figure is changed, working in the digital environment can influence the method used and, therefore, the associated visualization mode.

Assessing in a dynamic geometry environment is another alternative. Construction in this environment resembles paper-and-pencil assessment less (Petitfour, 2017), but this type of software helps teachers detect the construction approach used by identifying the properties of the constructed elements. For example, in the exercise shown in Figure 4, the teacher can detect whether the center of the circle was placed randomly by the pupil or whether it was placed according to figure's properties. If it is preferable to preserve tasks proposed in the paper-and-pencil assessment, it would be necessary to adapt the criteria to this new assessment mode.

Finally, a fifth possibility is assessment in an environment where pupils give the commands for tracings rather than producing tracings themselves. This would give more focus to the steps of the solution than

on the tracings, which seems sufficient to determine a pupil's visualization mode. This option is close to the dyad work system envisaged by Petitfour (2017), where one child simply gives another child construction instructions, which removes the distraction of manipulations and organizational tracings (Houdement & Petitfour, 2022).

### ***The Second Study: Assessment of Spatial Visualization in 3D Geometry***

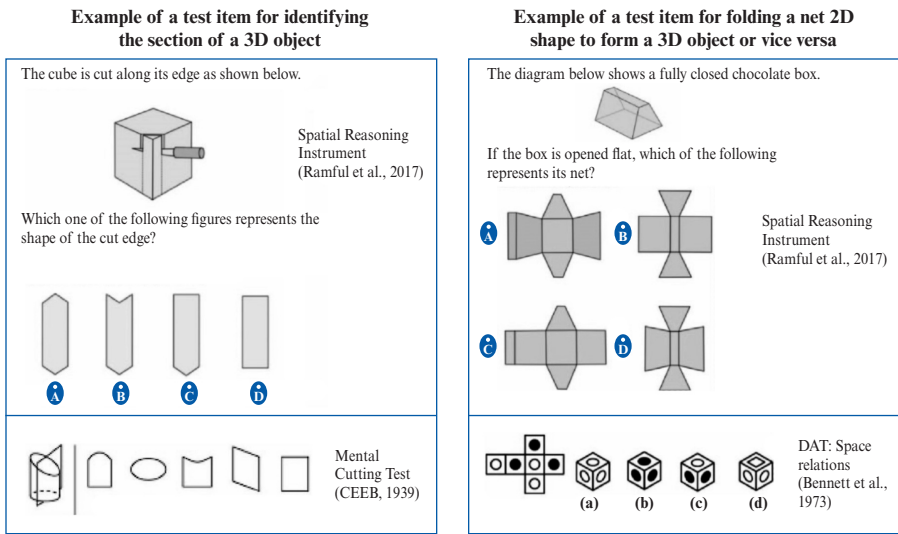
The second study focuses on the assessment of the 3D spatial visualization ability with pupils aged 8 to 15 years. Whilst paper-and-pencil tests exist to assess the level of development of this spatial ability (Ramful et al., 2017), these tests seem to have several limitations. After highlighting limitations associated with paper-and-pencil tests, we go on to describe the experiment conducted, allowing testing of a tool likely to overcome these limitations.

#### ***The Spatial Visualization Ability in 3D Geometry***

Like other spatial abilities, spatial visualization is a fundamental component of 3D geometry (Gutiérrez, 1992). It allows mental images of objects to be created/generated and manipulated/modified (Marchand, 2006), from a fixed or changing perspective on objects, to reason about objects and actions performed on objects, when objects are both visible and not visible (Battista et al., 2018). Strong understanding of concepts in the domain is required (Kaur et al., 2018) and is a strong predictor of pupil performance (Pittalis & Christou, 2010). Assessing development of this spatial ability in 3D geometry helps to understand certain difficulties encountered in learning, particularly for identifying difficulties in understanding abstract space (Duroisin, 2015; Duroisin & Demeuse, 2016). As a result, interventions can be envisaged to develop this ability and promote geometry learning, particularly in secondary education where recourse to abstract thinking is a dominant component of school content (Duroisin & Demeuse, 2015). Moreover, the pertinence of this assessment reaches beyond teaching and learning geometry, as the ability is indicative of success for other domains (Nagy-Kondor, 2014).

Several paper-and-pencil psychometric tests from the field of cognitive-developmental psychology assess spatial visualization ability in 3D space with different types of tasks. See Figure 5 for an example of two tasks: The first, identifying shapes of sections of 3D objects; and the second, folding a net 2D shape to form a 3D object or vice versa (Figure 5).

Figure 5  
*Extracts from Psychometric Tests Assessing Spatial Visualization*

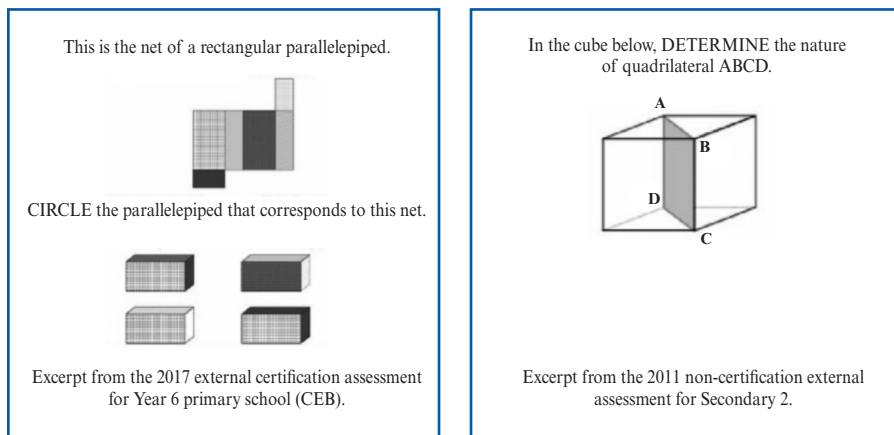


Also, external certification and non-certification assessments at school also propose questions similar to items in psychometric assessments (see Figure 6). This illustrates the important link between this ability and learning in 3D geometry, confirming the importance of its assessment, particularly at the end of primary or the beginning of secondary education.

However, this type of test has an important limitation due to the representation of 3D objects which is inherent to the medium. In paper-and-pencil tests, 2D representations of solids do not always provide properties that allow students to perceive the solid required. Many research studies, particularly in mathematics education (e.g., Kondo et al., 2014), have highlighted difficulties amongst pupils in reading representations. It therefore appears that the capacity to perceive solids represented in 2D bias the results supposed to assess visualization ability. Indeed, a pupil cannot mentally act on a represented solid if they do not perceive it correctly. This is therefore a limitation of paper-and-pencil tests which could be solved by providing pupils with the essential properties on the 2D representation to help them make the transition to the solid. However, this solution requires geometric knowledge and theoretical thinking, and as a result, the assessment would no longer focus exclusively on spatial visualization ability.

Figure 6

*Extract from National External Certification or Non-certification Assessments of the Wallonia-Brussels Federation Assessing 3D Spatial Visualization Ability*



The technological developments present another possibility which envisaged during the experiment: Presenting pupils with virtual representations of solids in a virtual  $2\frac{1}{2}$ D space (according to Bertolo, 2013), between the sensible 3D space and the graphical 2D space. Using dynamic information processing to simulate 3D space is an alternative to drawings of static characters.

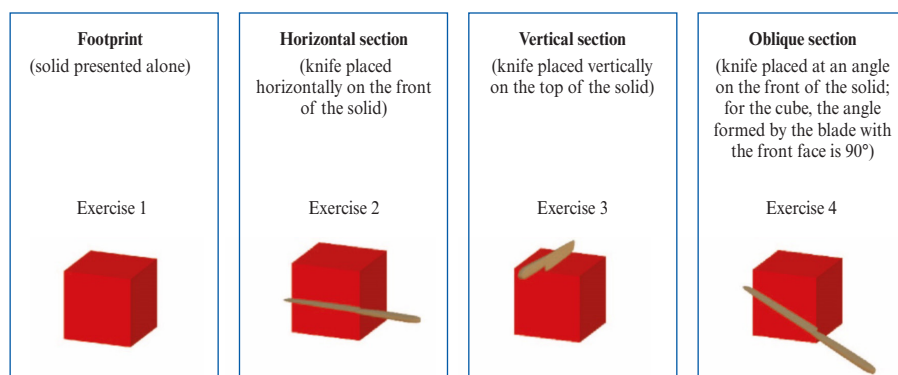
### *Description of the Test Assessing 3D Spatial Visualization*

The experiment, adapted from Duroisin (2015), aims to understand how spatial visualization develops in virtual environments amongst children and adolescents aged 8 to 15 years. From the fields of cognitive and developmental psychology, this experiment is part of a larger research project aimed at understanding the development of visuospatial and visuperceptive abilities and intends to further reflection on learning and assessment media used in 3D geometry.

During this experiment, two types of tasks were used to assess spatial visualization ability: Identifying shapes of footprints and sections of solids. The tasks were presented to a sample of 90 pupils distributed equally by age groups. The test was organized in a group session for approximately 40 minutes. Pupils used their tablets to individually observe each exercise and were provided with a response booklet.

First, the experimenter explained the notions of footprint and section with two familiarization exercises. They ensured the pupils understood the expected results of drawing the shape of the footprint for footprint exercises and drawing the shape of the surface of the section made by the knife if it continues its trajectory for the section exercises. A total of 20 exercises were proposed to the pupils. They were questioned on five solids in the following order: cube, cone, sphere, cylinder, and ring (torus). For each solid, there was a footprint exercise then three section exercises: a transverse section exercise, a longitudinal section exercise and an oblique section exercise<sup>2</sup>. Figure 7 shows the first four exercises for a cube and Annex 2 shows all the exercises.

Figure 7  
*Items for a cube*



In each footprint exercise, pupils were presented a 2½D representation of the solid on the tablet in parallel perspective. A video of approximately 45 seconds showed the solid rotating on itself<sup>3</sup> so the pupil was able to observe it in different positions. At the end of the video, the photograph of the initial view of the solid was presented with the instruction (Figure 8).

2. The terms ‘transverse’, ‘longitudinal’ and ‘oblique’ which describe different sections are used with reference to Duroisin (2015).
3. Following an earlier experiment designed to assess perceptual skills, all pupils had already viewed the video of the solid.

The pupil was asked to draw freehand the shape of the footprint of the solid in the space provided in the paper response booklet, and, if possible, to write the name of the shape.


Figure 8  
*Instruction After Observing the Video*

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**EXERCISE 1:**

Find the shape of the solid's footprint.  
On the Exercise 1 page:

- Draw the shape of the solid's footprint.
- Write (if possible) the name of the shape drawn.



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In each of the section exercises, pupils were presented with a 2½D representation of the solid and the knife on the tablet. Like the footprint exercises, a video was shown, but this time, the knife and the solid rotate together allowing the pupil to fully perceive the position of the knife in relation to the solid. The pupil was then asked to draw freehand the shape of the surface of the proposed section and, if possible, to write the name of the drawn shape.

***Discussion Relating to the Assessment: Strengths, Limitations and Alternative***

Paper-and-pencil tests assessing spatial visualization ability in 3D geometry (e.g., Ramful et al., 2017) require very little materials and are easy to conduct in groups, however, they can not be applied to pupils unable to adequately perceive solids represented in 2D. This is a genuine difficulty for certain learners (Camou, 2012; Kondo et al., 2014). If these tests are used without first verifying the pupils can correctly perceive the solids represented in 2D, the results are potentially biased. It would be impossible to differentiate between a pupil's inability to demonstrate spatial visualization and their inability to perceive the solid from its representation. This is particularly the case for solid section tests, where pupils need to adequately perceive both the solid and the way the knife is placed on the solid to imagine the correct section, which can be difficult in 2D

representation. This limitation, already raised by several authors such as Hawes et al. (2015) and Rahe and Quaiser-Pohl (2022), exceeds the scope of assessing cognitive processes, and applies to any assessment of learning in 3D geometry featuring this type of representation.

For this assessment, virtual material is suggested as a replacement for paper and pencils. Many paper-and-pencil psychometric tests for assessing cognitive processes have already been adapted into digital format (Parsey & Schmitter-Edgecombe, 2013), as this transformation offers various advantages such as consistency in administration and scoring, precision in the administration of stimuli, and better pupil participation and engagement (Bauer et al., 2012; Cerrato & Ponticorvo, 2017). Nevertheless, in most cases, digitized paper-and-pencil assessments still represent solids in 2D, not taking advantage of digitization to propose 2½D representations, therefore, the associated limitations persist. Yet, 2½D representations offer dynamic information processing that gives the impression of three-dimensional vision (Bako, 2003), thus overcoming the limitations of paper-and-pencil tests. Indeed, previous experiments have shown that more and a majority of pupils perceive virtual representations of solids correctly than in the case of 2D representations (Beuset & Duroisin, 2024b).

This assessment allowed identification of diversity in the pupils' errors, some of which are easily interpreted and therefore clearly understood. This is the case of the typical errors illustrated in Figure 9, which show that certain pupils have difficulties taking 3D into account. This can be associated with what Parzysz (1988) calls the conflict between what is seen and what is known.

Other typical errors produced by pupils are more difficult to explain. They indicate, but do not explain, a deficit in spatial visualization capacity, making it difficult to support pupils' development of the ability. This is another limitation in the study of such a process but is not specific to assessment with virtual representations. Another difficulty is the interpretation of pupils' freehand drawings, for example when differentiating a circle from an oval. Asking pupils to name their production can remove the ambiguity, but this solution is limited because children and adolescents do not necessarily master geometric knowledge. The object of the study, with focus on the drawing, is the child's spatial knowledge only. This also leads to cases where the drawing and the associated name do not seem to correspond, as illustrated in Figure 10. Asking pupils to draw with instruments rather than freehand drawings may be useful for adolescents but seems poorly adapted to younger pupils given their difficulty using instruments. Also,

pupils with particular profiles, such as dyspraxic children who may have kinesthetic difficulties, are disadvantaged regardless of drawing freehand or with instruments. To avoid these limitations, production of drawings could be replaced with recognition tasks, such as multiple-choice questions. However, this mode entails new limitations (e.g., influence of inhibition capacity, reasoning facilitated by propositions, etc.) and does not generate the same diversity of typical errors observed in the production task.

Figure 9  
*Typical Errors Observed*







Exercises	Typical Errors	Interpretation
<p data-bbox="173 644 328 666">Cube: oblique section</p> 		<p data-bbox="598 647 1014 913">The pupil's answer is the simulated outline of the cut on the face on which the knife is placed. The pupil has difficulty taking the 3D into account because they juxtapose the shape of the cut (the shape accessible inside the solid) with the image of the section seen from the outside. This could also be due to the task being misunderstood, even though the two familiarization exercises provided should have ensured that section concept was understood.</p>
<p data-bbox="176 954 324 996">Cylinder: horizontal section:</p> 		<p data-bbox="598 960 1014 1086">The pupil's answer is the shape of the section as it appears on the representation of the solid. The pupil therefore has difficulty taking into account the 3D aspect, due to a deficit in the perception of the solid itself.</p>

Figure 10  
*A Pupil's Production Showing Inconsistency Between Drawing and Name*

<p data-bbox="184 1367 385 1390">Cylinder: horizontal section</p> 	<p data-bbox="552 1367 621 1390">Drawing:</p> 	<p data-bbox="770 1367 1017 1413">Name associated with the drawing by the pupil: "circle"</p>
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Working memory load due to the presentation mode of solids is another limitation. The use of virtual solids requires mental 3D reconstruction (Vivian et al., 2014) which increases the mental load (Ayres & Paas, 2009). An overload in working memory can affect pupils' performances, especially as the cognitive load required by spatial tasks such as sections is already high (Hawes & Ansari, 2020). Other executive functions, besides working memory, can also influence performance and therefore bias the study of the targeted spatial ability. For example, a deficit in cognitive flexibility can influence pupils' performances on the visualization task (Beauset et al., 2025).

Whilst digital assessment offers added value in comparison to paper-and-pencil assessment, the use of such media can also have an impact on pupils' performances and, consequently, partially bias the assessment. In this sense, the use of tangible material to present 3D objects appears more suitable. This is the assessment mode proposed by Hawes et al. (2015) for assessing children's spatial ability for mental rotation. Although it may be more complex to organize, particularly for organizational and material reasons, this mode avoids difficulties in 3D perception and overloading the working memory.

### ***The Challenges, Limitations and Options Relating to the Assessment of Cognitive Processes in Geometry***

Vergnaud (1989) and other scholars posited that studying pupils' cognitive functioning is essential for teaching. Understanding and assessing the development of cognitive processes in children and adolescents is an important issue. Such assessment promotes understanding certain pupils' learning difficulties and the implementation of adapted interventions for developing targeted competences. This article has focused on the assessment of one of these processes, namely spatial visualization in 2D and 3D geometry.

Whilst paper-and-pencil tests exist to assess this process, they have many limitations which make analysis complex, and as a result, this assessment remains challenging.

First, the use of certain cognitive processes governing the solution of tasks to assess the investigated ability, which is observed in both 2D and 3D geometry. The tests analyzed in this article, like most assessments, engage transversal cognitive processes including executive functions such as working memory and cognitive flexibility. The development of these

executive functions influences pupils' performances on the proposed tasks. As a result, it is difficult to interpret performance, particularly in the case of failure, which may be due to a deficit of executive functions rather than difficulty in demonstrating visualization.

Beyond these transversal cognitive processes, other elements, specific to geometry, are also likely to affect pupils' performances on tasks assessing visualization. For example, in 3D geometry, it is difficult to dissociate the paper-and-pencil assessment used for visualization from the ability of perception of 3D. This limitation can moreover relate to any paper-and-pencil assessment in connection with 3D geometry, whether for assessing learning (e.g., external certification assessments) or assessing cognitive processes (e.g., psychometric tests of mental rotation in 3D). A task starting with a 2D representation of a solid requires a pupil to adequately perceive the represented solid. A perception problem can therefore affect the pupil's results. It is important to recognize this limitation, as research, particularly in geometry education, has long shown that seeing in space can cause difficulties for pupils and that reading 2D representation can be difficult. In addition to visual capacities, motor capacities can also have an impact on pupils' performances, as may be the case in the task used to assess visualization in plane geometry. This is inherent to the complexity of geometry which requires complete cognitive activity, particularly involving vision and gestures (Duval, 2005). Finally, other elements can also influence pupils' performances during assessment, such as the didactic contract, as shown above in 2D geometry.

Another difficulty encountered in the studies conducted concerns the interpretation of the level of development of visualization from certain pupils' productions. We highlighted the fact that traces left by pupils in the 2D geometry visualization assessment suggest, but without certitude, that they used certain cognitive processes, particularly when they leave no or very few traces (see Figure 4). Whereas, in the assessment in the second study, we observed difficulty in understanding the origin of certain typical errors made by pupils, making it difficult to determine whether the pupil is indeed demonstrating visualization or not.

The limitation relating to the assessment of pupils with particular profiles, particularly dyspraxic pupils, persists. At a time when inclusion is a general concern, the assessment of cognitive processes amongst these pupils is an important issue, especially as this population is likely to have more learning difficulties in geometry.

Through the two studies and implementation of the two tools presented, we highlighted the fact that digital tools can counter some of the limitations encountered. However, these solutions present other limitations such as the need for equipment, need for user familiarization, and potential impact on performance. For example, as the first study illustrates, traces of the steps in the solution applied and retained provide for better interpretation of the cognitive processes used. Moreover, digital tools may also be a solution for 3D geometry tests, as the medium facilitates the perception of solids. Nevertheless, caution must be exercised regarding the use of these technologies. Future research would benefit from experiments to identify whether pupils' performances on proposed visualization tasks are influenced by the different testing modes. Such an approach would allow verification of Rose and Foreman's hypothesis (1999) that cognitive processes differ between real and virtual environments.

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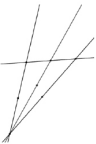

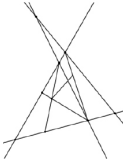





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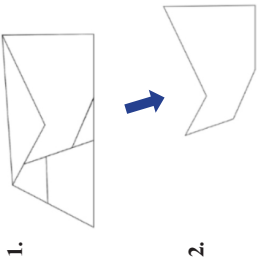


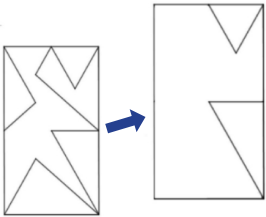
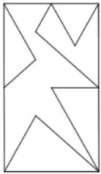
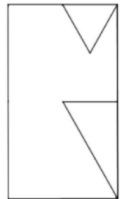
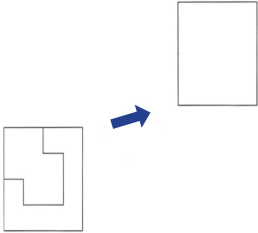
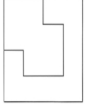

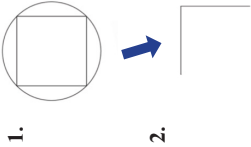
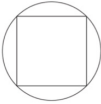

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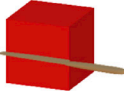







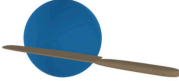

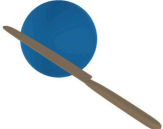




*Annex A – Exercises from the Test Assessing Visualization Mode (Iconic/Non-iconic)*

<p>Identical restoration of a configuration of secant lines starting from a given set of points, enabling direct reproduction</p>	<p>Identical restoration of a configuration of intersecting lines starting from a given set of points, enabling indirect reproduction</p>	<p>Identical restoration of a figure when the identification of alignments is sufficient</p>	<p>Identical restoration of a figure for which the identification of alignments is not sufficient or a transfer of measurements is necessary</p>
<p><b>REPRODUCE</b> exactly drawing 1, using drawing 2 as a starting point. <b>All lines and points must appear.</b> Use the instruments of your choice. Your lines must be precise. Leave the traces of your construction</p>	<p><b>REPRODUCE</b> exactly drawing 1, using drawing 2 as a starting point. <b>All lines and points must appear.</b> Use the instruments of your choice. Your lines must be precise. Leave the traces of your construction</p>	<p><b>REPRODUCE</b> drawing 1 exactly, using drawing 2 as a starting point. Use the instruments of your choice. Your lines must be precise. Leave the traces of your construction.</p>	<p><b>REPRODUCE</b> drawing 1 exactly, using drawing 2 as a starting point. Use the instruments of your choice. Your lines must be precise. Leave the traces of your construction.</p>
 <p>1.</p>  <p>2.</p>	 <p>1.</p>  <p>2.</p>	 <p>1.</p>  <p>2.</p>	 <p>1.</p>  <p>2.</p>





*AAnnex A – Exercises from the Test Assessing Visualization Mode (Iconic/Non-iconic) (suite)*

Identical restoration of a complex figure	Restoration with enlargement of a complex figure	Identical restoration of a figure constructed from a background grid (set of perpendicular and parallel lines)	Identical restoration of a figure with a circle inside
<p><b>REPRODUCE</b> drawing 1 exactly, using drawing 2 as a starting point. Use the instruments of your choice. Your lines must be precise. Leave the traces of your construction.</p>  <p>1.  </p> <p>2.</p>	<p><b>REPRODUCE</b> drawing 1 exactly, using drawing 2 as a starting point. Use the instruments of your choice. Your lines must be precise. Leave the traces of your construction.</p>  <p>1.  </p> <p>2.</p>	<p><b>REPRODUCE</b> drawing 1 exactly, using drawing 2 as a starting point. Use the instruments of your choice. Your lines must be precise. Leave the traces of your construction.</p>  <p>1.  </p> <p>2.</p>	<p><b>REPRODUCE</b> drawing 1 exactly, using drawing 2 as a starting point. Use the instruments of your choice. Your lines must be precise. Leave the traces of your construction.</p>  <p>1.  </p> <p>2.</p>

**Annex B – Exercises from the Test Assessing 3D Spatial Visualization Ability**

	<b>Footprint</b> (solid presented alone)	<b>Transverse section</b> (knife placed horizontally on the front of the solid)	<b>Longitudinal section</b> (knife placed vertically on the top of the solid)	<b>Oblique Section</b> (knife placed at an angle on the front of the solid; for the cube, the angle formed by the blade with the front face is 90°)	
<b>Solids</b>					
	<b>Cube</b>	<i>Exercise 1</i>	<i>Exercise 2</i>	<i>Exercise 3</i>	<i>Exercise 4</i>
					
	Expected answer: a square	Expected answer: a square	Expected answer: a square	Expected answer: a rectangle	
	<b>Cone</b>	<i>Exercise 5</i>	<i>Exercise 6</i>	<i>Exercise 7</i>	<i>Exercise 8</i>
					
	Expected answer: a circle	Expected answer: a circle	Expected answer: an isosceles triangle	Expected answer: an oval	
	<b>Sphere</b>	<i>Exercise 9</i>	<i>Exercise 10</i>	<i>Exercise 11</i>	<i>Exercise 12</i>
					
	Expected answer: a dot or a small circle (max. diameter 0.5 cm)	Expected answer: a circle	Expected answer: a circle	Expected answer: a circle	
	<b>Cylinder</b>	<i>Exercise 13</i>	<i>Exercise 14</i>	<i>Exercise 15</i>	<i>Exercise 16</i>
					
	Expected answer: a circle	Expected answer: a circle	Expected answer: a rectangle	Expected answer: an oval	

**Annex B – Exercises from the Test Assessing 3D Spatial Visualization Ability (suite)**

	<b>Footprint</b> (solid presented alone)	<b>Transverse section</b> (knife placed horizontally on the front of the solid)	<b>Longitudinal section</b> (knife placed vertically on the top of the solid)	<b>Oblique Section</b> (knife placed at an angle on the front of the solid; for the cube, the angle formed by the blade with the front face is 90°)
<b>Ring</b>	<i>Exercise 17</i> 	<i>Exercise 18</i> 	<i>Exercise 19</i> 	<i>Exercise 20</i> 
	Expected answer: one circle or two concentric circles very close together (max. 0.5 cm apart)	Expected answer: two concentric circles	Expected answer: two separate circles	Expected answer: two separate ovals